An Introduction to Depth of Field

By Jeff Conrad

In many types of photography, it is desirable to have the entire image sharp. Strictly speaking, this is impossible: a camera can precisely focus on only one plane; a point object in any other plane is imaged as a disk rather than a point, and the farther a plane is from the plane of focus, the larger the disk. This is illustrated in Figure 1.

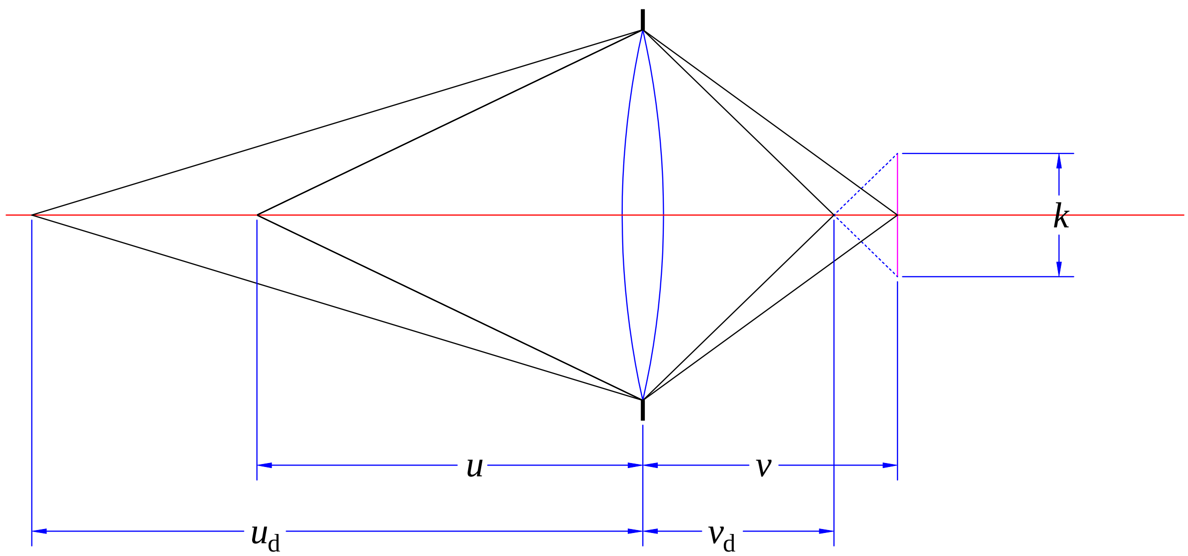

Figure 1. In-Focus and Out-of-Focus Objects

Following convention for optical diagrams, the object side of the lens is on the left, and the image side is on the right. The object in the plane of focus at distance u is sharply imaged as a point at distance v behind the lens. The object at distance u_d would be sharply imaged at a distance v_d behind the lens; however, at the focus distance v, it is imaged as a disk, known as a *blur spot*, of diameter k.

Reducing the size of the aperture stop reduces the size of the blur spot, as shown in Figure 2. If the blur spot is sufficiently small, it is indistinguishable from a point, so that a zone of acceptable sharpness exists between two planes on either side of the plane of focus. This zone is known as the *depth of field* (DoF). The plane at u_n is the near limit of the DoF, and the plane at u_f is the far limit of the DoF. The diameter of a “sufficiently small” blur spot is known as the *acceptable circle of confusion*, or simply as the *circle of confusion* (CoC).

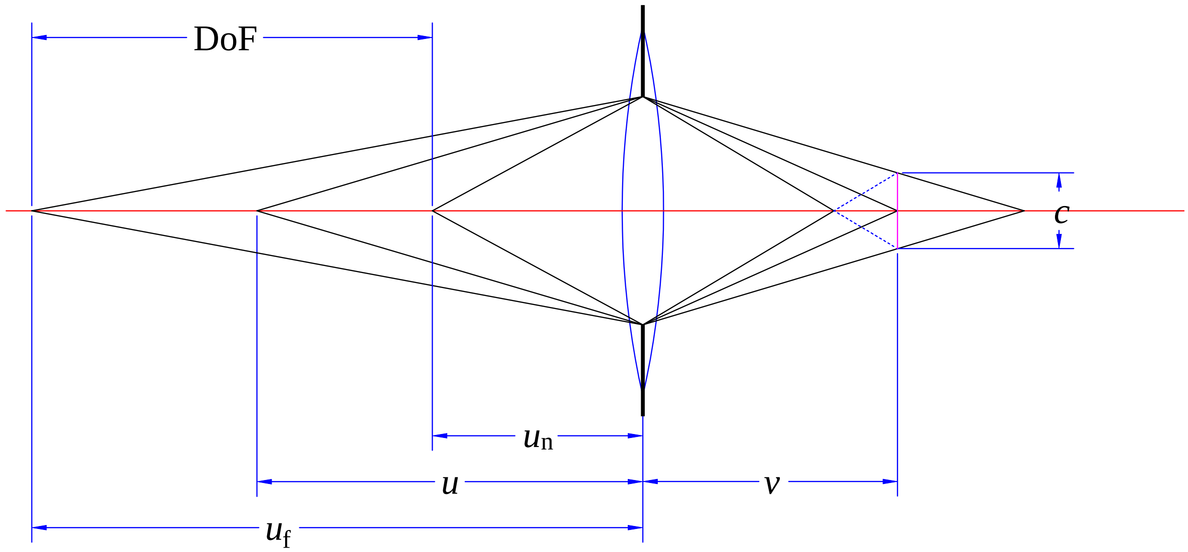

Figure 2. Depth of Field and Circle of Confusion

The DoF depends on the CoC, the lens focal length, the object distance, and the lens *f*-number. To a first approximation, the lens focal length and object distance determine the image magnification, so that DoF depends on magnification and *f*-number; lesser magnification and greater *f*-number (smaller lens opening) give a greater DoF.

The CoC is somewhat subjective, and depends on several factors, including viewing conditions and required enlargement. Standard assumptions usually are an 8″×10″ final image viewed at a distance of 250 mm, and that a final-image CoC of 0.2 mm at that distance cannot be distinguished from a point. If the original image is smaller than 8″×10″, it must be enlarged to make 8″×10″ final image, and the CoC in the original image is reduced by the required enlargement. For example, a 4×5 image must be enlarged 2×, so its standard CoC is 0.1 mm. Although standard values often are fine, some adjustment may be needed if the planned viewing conditions differ from those assumed in determining the standard values. This may be especially true if an 8″×10″ final image will be made from a 35 mm image, where only a 5× enlargement often is assumed. Adjustment also may be required if the photographer intends to examine large prints at close distances with a magnifying glass. However, because of diffraction, the practical amount of adjustment is limited when substantial DoF is required.

Except for camera movements, there only are two DoF-related tasks: setting the focus and the lens *f*-number. This paper concentrates on the few formulae needed to accomplish those tasks; the derivations of those formulae and many others are given in Conrad (2006).

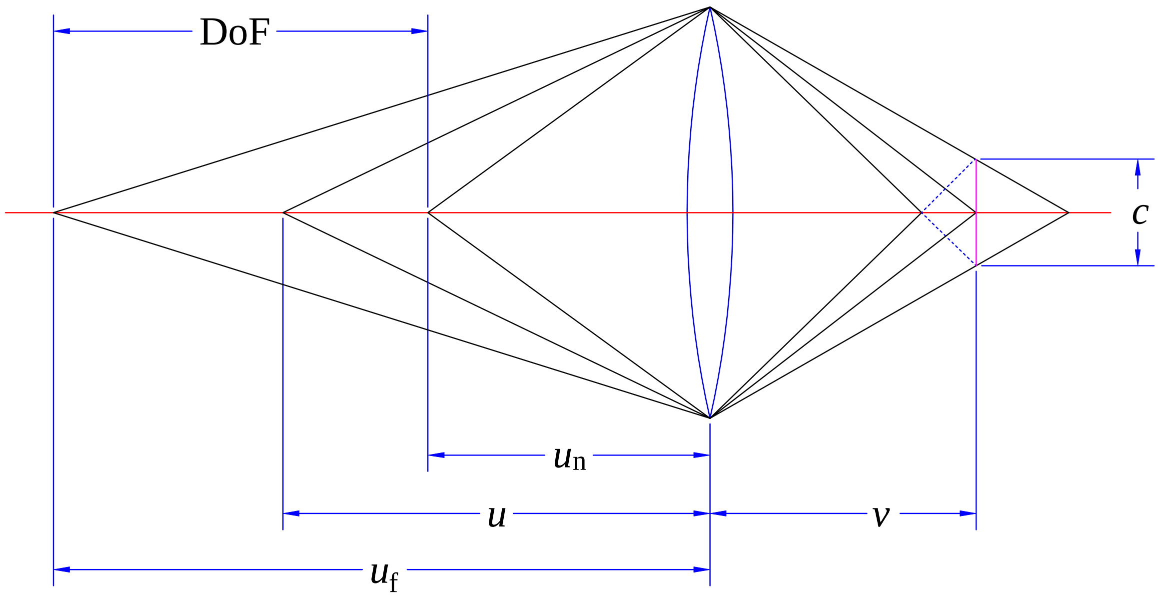

Figure 3. Object Distances and DoF

Object-Side Relationships

Object-side relationships can be useful for illustrating DoF concepts, and possibly for hand-camera users working with lens distance scales, although it usually is easier to use lens DoF scales. With a view camera, it nearly always is easier to use the image-side relationships, discussed in the next section.

The relevant quantities are shown in Figure 3. The distances indicated by u_x on the left side are the object distances; the image distance v on the right corresponds to the focused object at distance u.

More often than not, the photographer will choose the limits of DoF, and set the focus and f-number accordingly. To have the DoF between near and far distances u_n and u_f,

$$\text{focus} = \frac{2 \times \text{near distance} \times \text{far distance}}{\text{near distance} + \text{far distance}},$$

or

$$u = \frac{2u_\mathrm{n}u_\mathrm{f}}{u_\mathrm{n} + u_\mathrm{f}} \qquad (1)$$

The f-number is

$$N = \frac{f^2}{c} \frac{u_\mathrm{f} - u_\mathrm{n}}{u_\mathrm{f}(u_\mathrm{n} - f) + u_\mathrm{n}(u_\mathrm{f} - f)}, \qquad (2)$$

where N is the f-number, f is the lens focal length, and c is the CoC. The difference $u_\mathrm{f} - u_\mathrm{n}$ is the DoF. This is a bit tedious to calculate, but if the near and far limits of DoF are large in comparison with the lens focal length,

$$f\text{-number} \approx \frac{(\text{focal length})^2 \times (\text{far distance} - \text{near distance})}{\text{CoC} \times \text{far distance} \times \text{near distance}},$$

or

$$N \approx \frac{f^2}{c}\,\frac{u_\mathrm{f} - u_\mathrm{n}}{2u_\mathrm{f}u_\mathrm{n}} \tag{3}$$

This still is rather tedious, but as will be seen, Eqs. (1) or (3) hardly ever are necessary. Eq. (3) is instructive in that it shows the reciprocal relationship between N and c: for constant DoF, increasing the *f*-number is equivalent to decreasing the CoC by the same ratio; this can be useful with a hand-camera lens whose DoF scales are based on a CoC that is not appropriate for the intended viewing conditions. For example, if a 35 mm camera has DoF scales determined for a CoC of 0.035 mm and viewing conditions require a CoC of 0.025 mm, the smaller CoC can be achieved by using an *f*-number one step greater than indicated by the lens DoF scale. Note also that the distance given by Eq. (1) is independent of N: if the *f*-number is increased to decrease the effective CoC, there is no need to refocus.

When the far limit of DoF is infinity, and the *f*-number is fixed (possibly because of motion blur considerations), the focus distance is

$$u_\mathrm{h} = \frac{f^2}{Nc} + f \tag{4}$$

The distance u_h is called the *hyperfocal distance*. At the hyperfocal distance, a difference of one focal length is insignificant, so Eq. (4) often is given simply as

$$\text{hyperfocal distance} \approx \frac{(\text{focal length})^2}{f\text{-number} \times \text{CoC}},$$

or

$$u_\mathrm{h} \approx \frac{f^2}{Nc} \tag{5}$$

When the focus is set to the hyperfocal distance, the near limit of DoF is half the hyperfocal distance, or

$$u_\mathrm{n} = \frac{u_\mathrm{h}}{2} \approx \frac{f^2}{2Nc}, \tag{6}$$

so the DoF extends from half the hyperfocal distance to infinity. If the near limit of DoF is fixed,

$$\text{focus} = 2 \times \text{near distance},$$

or

$$u = 2u_\mathrm{n}; \tag{7}$$

the *f*-number is

$$f\text{-number} \approx \frac{(\text{focal length})^2}{2 \times \text{CoC} \times \text{near distance}},$$

or

$$N \approx \frac{f^2}{2cu_n} \tag{8}$$

The near:far ratio of the DoF is

$$\frac{\text{near DoF}}{\text{far DoF}} = \frac{\text{near distance}}{\text{far distance}},$$

or

$$\frac{u - u_n}{u_f - u} = \frac{u_n}{u_f} \tag{9}$$

Except when the near and far limits of DoF coincide, the focus point always is closer to the near limit. Despite long-standing legend, the near-to-total ratio is ⅓ only when $u_f = 2u_n$, so that the "rule" to focus ⅓ of the way into a scene hardly ever is valid.

In practice, it seldom is necessary to make any of these calculations. With a manual-focus hand camera whose lens includes a DoF scale, the settings can be made using that scale. With view camera, the required settings can be determined on the image side, where the calculations are much simpler. Unfortunately, controlling DoF with an autofocus hand camera usually is quite difficult with or without calculations.

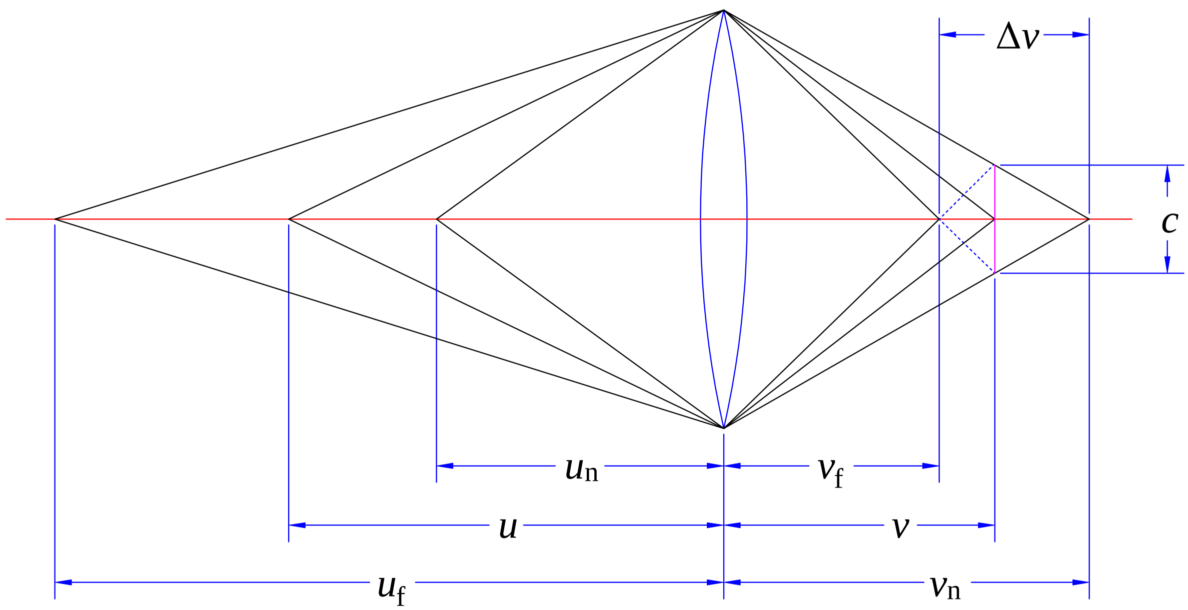

Figure 4. Image Distances and Focus Spread

Image-Side Relationships

In practice, camera settings usually are determined using image distances, whether directly in the case of a view camera, or by using distance and DoF scales on hand-camera lenses. The procedure with a view camera is quite simple: focus on the near object, and note the position of the standard; focus on the far object, and again note the position of the standard.

The relevant distances are shown in Figure 4: the image distances v_n and v_f are the positions of the standard when focused on the near and far objects, respectively; they correspond to object distances u_n and u_f. The difference $v_n - v_f$ between the two positions of the standard is the *focus spread* Δv.

In most cases,

$$\text{focus} \approx \text{image distance of far object} + \frac{\text{focus spread}}{2},$$

or

$$v \approx v_f + \frac{\Delta v}{2}; \qquad (10)$$

Focus is set to halfway between the near and far positions of the standard; it does not appear quite that way in Figure 4 because the focus spread is greatly exaggerated. When the focus spread is reasonably small, the *f*-number is

$$N \approx \frac{1}{1+m}\frac{\Delta v}{2c}, \qquad (11)$$

where m is the magnification of the focused object. Except at close working distances, m is small, and Eq. (11) often can be simplified to

$$f\text{-number} \approx \frac{\text{focus spread}}{2 \times \text{CoC}},$$

or

$$N \approx \frac{\Delta v}{2c} \qquad (12)$$

Note again the reciprocal relationship between N and c in Eqs. (11) and (12); for a given focus spread, using a greater *f*-number is equivalent to using a smaller CoC.

Eqs. (10) and (12) also apply if swings or tilts have been set. Some view cameras, such as those by Sinar, incorporate mechanical focus and DoF calculators, so that no manual calculations are needed.

If the camera does not include a DoF calculator, measurement of focus spread is much easier if the bed or focusing rail includes a scale, and measurements can be more precise if the focusing knob includes an additional scale. See Hayashi for a description of adding a Sinar-type DoF scale to the focusing knob, and Evens (2003) for a discussion of adding scales to both the rail and the knob.

Equations (10) and (12) are the basis for most hand-camera lens DoF scales, even though the scales indicate to subject-to-image distance. These equations also appear to have been the basis for Canon's Depth-of-Field AE mode, which, lamentably, was discontinued on models introduced after early 2004.

Maximum *f*-Number

The *f*-number given by Eq. (12) is the minimum that will give the specified sharpness at the limits of DoF. In many cases, sharpness at the DoF limits can be improved by using a greater *f*-number, although not without consequence: except with controlled lighting, a greater *f*-number requires a longer exposure, which eventually can result in motion blur.

Moreover, the benefit of less blur from defocus eventually is offset by *diffraction*, the bending of light as it passes through an aperture, causing the light to spread slightly and produce a softer image. Diffraction increases as the *f*-number is increased; it is unavoidable with any lens of any design, and affects the center of the image as well as the limits of DoF. Once the effect of diffraction equals that from defocus, any additional increase in *f*-number will result in less sharpness at the limits of DoF as well as at the plane of focus. For practical purposes, that limit can be given as[1]

$$\text{maximum } f\text{-number} = \sqrt{400 \times \text{focus spread}}\,,$$

or

$$N_{\max} = \sqrt{400\,\Delta v}\,, \tag{13}$$

where Δv is the focus spread in mm. For most reasonable values of focus spread, the *f*-number given by Eq. (13) is greater than that given by Eq. (12), so Eqs. (12) and (13) can be regarded as limits for an acceptable range of *f*-numbers. In many cases, considerations of motion blur will determine the selection the *f*-number within the acceptable range. For example, for a 4×5 camera, the standard CoC is 0.1 mm; with a 4 mm focus spread, the conventional *f*-number from Eq. (12) is

$$N = \frac{4}{2 \times 0.1} = 20$$

The maximum *f*-number from Eq. (13) is

$$N_{\max} = \sqrt{400 \times 4} = 40\,,$$

so the actual *f*-number should be somewhere between 20 and 40.

The *f*-number from Eq. (13) is the optimum value for the DoF limits, but it may not be optimal for the entire image. Most large-format lenses are sharpest between *f*/16 and *f*/22, and increasing the *f*-number beyond this always reduces sharpness for objects in the plane of focus. In many cases, reasonably uniform sharpness throughout the image is preferable to optimal sharpness in the plane of focus, but the decision may depend on the individual image, and ultimately, is an aesthetic judgment that must be made by the photographer. Using an *f*-number greater than that given by Eq. (13) *never* increases sharpness. It can be argued that if the blur is less than the threshold of detection, any increase in sharpness is irrelevant; however, having the greatest possible sharpness can be useful if at some time it is decided to make a larger final image than originally planned.

Minimum and Maximum *f*-Numbers on Hand-Camera Lenses

With a hand camera, focus spread usually is difficult to measure, but if the lens manufacturer's CoC is known, the maximum *f*-number can be determined from the *f*-number marked on the lens as follows:

$$\text{maximum } f\text{-number} = \sqrt{800 \times \text{CoC} \times \text{marked } f\text{-number}}$$

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[1] The basis for Eq. (13) is given in Conrad (2006). Using considerably different methods, Hansma (1996) developed a nearly identical formula: $N_{\text{opt}} = \sqrt{375\,\Delta v}$.

If the CoC used to determine the len's marked *f*-numbers is not appropriate for the intended viewing conditions, appropriate minimum *f*-numbers as well as maximum *f*-numbers must be calculated. For example, if a 35 mm camera lens's marked *f*-numbers are based on a 0.035 mm CoC, and the intended conditions require a CoC of 0.025 mm, the appropriate minimum *f*-numbers are approximately one exposure step greater than those marked. The maximum *f*-numbers then are determined from

$$\text{maximum } f\text{-number} = \sqrt{28 \times \text{marked } f\text{-number}}$$

The results are shown in Table 1; all values are rounded to the nearest ⅓ step.

Table 1. Marked, Minimum, and Maximum *f*-Numbers

$N_{0.035}$	$N_{0.025}$	N_{max}
1.4	2	6.3
2	2.8	7.1
2.8	4	9
4	5.6	10
5.6	8	13
8	11	14
11	16	18
16	(22)	20
22	(32)	25
(32)	(45)	29

The subscripts in the first two column headings indicate the CoC used in determining the *f*-numbers. The marked *f*-numbers, based on a 0.035 mm CoC, are shown in the first column. The minimum *f*-numbers, based on a 0.025 mm CoC, are shown in the second column, and the maximum *f*-numbers are in the last column. The *f*-numbers in parentheses, beginning with 32 in the first column and 22 in the second column, are greater than the maximum *f*-numbers; when DoF measurements indicate an *f*-number in parentheses, the maximum *f*-number should be used instead. Eq. (13) is independent of focal length, so the third column of Table 1 is valid for any lens based on the same CoC.

For example, if measurements indicated a required *f*-number of 5.6 on the lens DoF scale, the minimum *f*-number would be 8, and the maximum would be 13, so the range of acceptable *f*-numbers would be from 8 to 13. If the indicated *f*-number were 16, the minimum *f*-number would be 22 and the maximum would be 20; accordingly, the maximum should be used. When the minimum *f*-number is greater than the maximum, the blur spot will be larger than that specified by the 0.025 mm CoC. It simply is not always possible to achieve the desired sharpness when great DoF is required.

If the lens manufacturer's CoC is not known, it often can be estimated from the lens distance and DoF scales by setting the infinity mark on the distance scale opposite the greatest marked *f*-number on the DoF scale; the focus index mark then is at the hyperfocal distance. Solving Eq. (4) for c gives

$$c = \frac{f^2}{N\left(u_{\mathrm{h}} - f\right)}$$

Lens distance scales usually indicate object-to-image distance. At the hyperfocal distance, the indicated distance can be taken as the object distance + focal length with minimal error; if x_h is the indicated hyperfocal distance,

$$\mathrm{CoC} \approx \frac{(\text{focal length})^2}{f\text{-number} \times (\text{indicated distance} - 2 \times \text{focal length})},$$

or

$$c \approx \frac{f^2}{N(x_\mathrm{h} - 2f)} \tag{14}$$

References

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